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NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 17

Interest Rate Risk

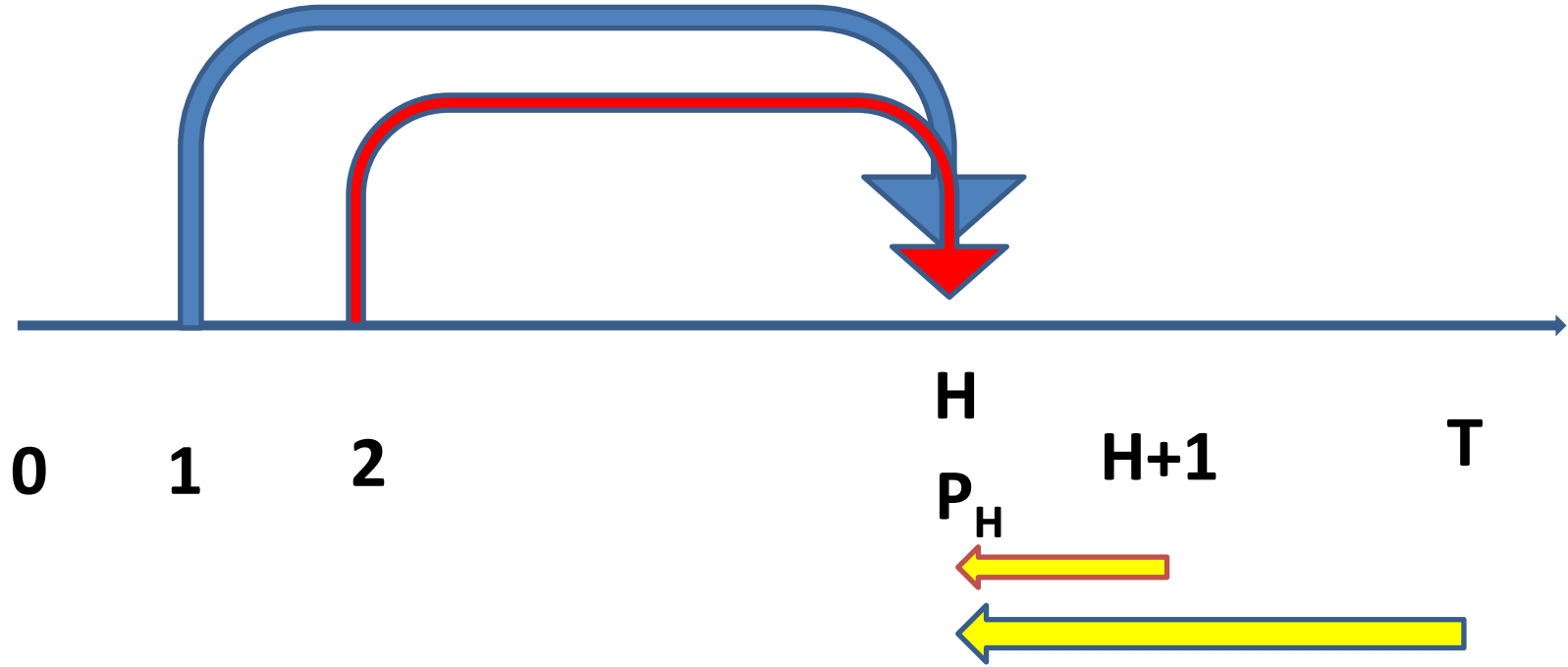
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INTEREST RATE RISK





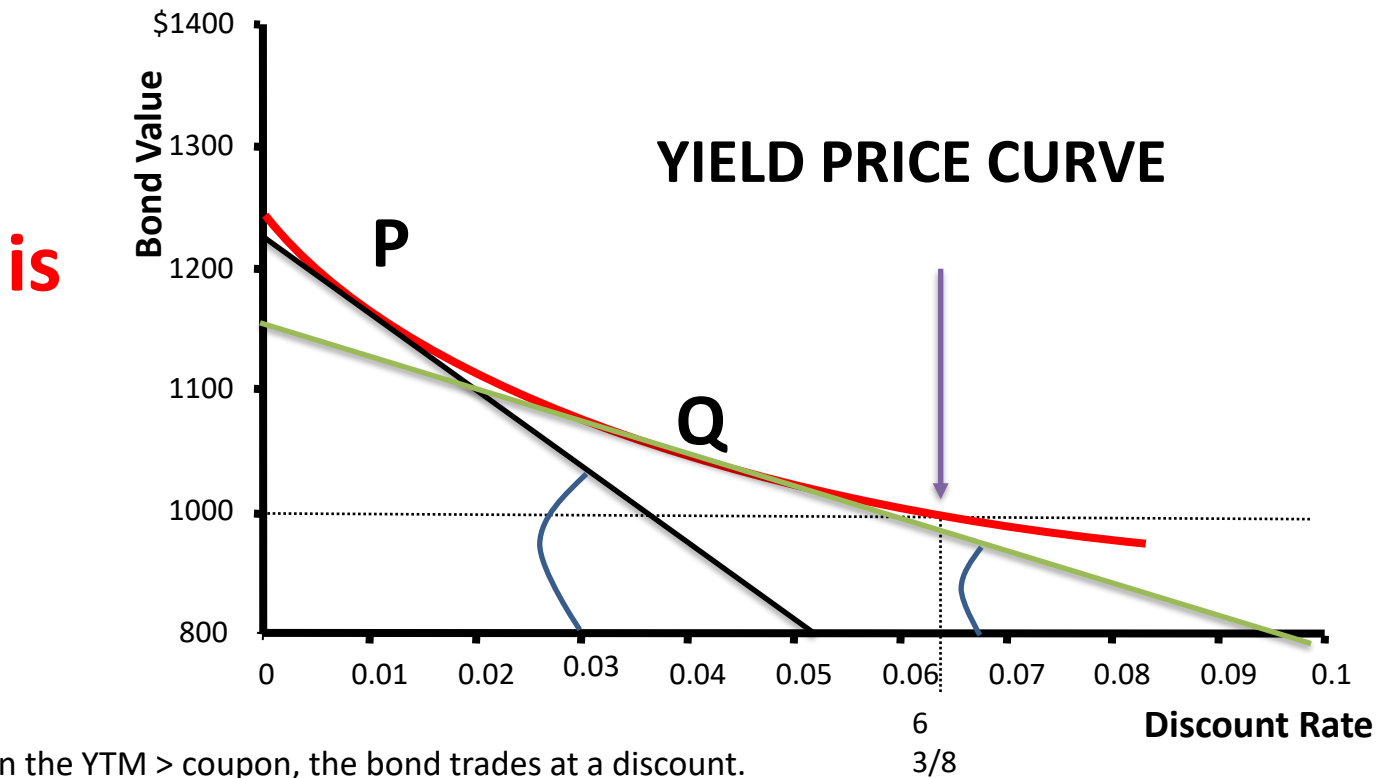
INTEREST RATE RISK

- Since the price of a bond fluctuates with market interest rates, the risk that an investor faces when investing in a bond portfolio is that:
- (i) the price of the bond at end of holding period will decline if market interest rates rise **UNANTICIPATEDLY**. This risk is referred to as price risk.
- (ii) the income from reinvestment of coupons may not yield the desired return if interest rate fall. This is called reinvestment rate risk.



YTM AND BOND PRICE

**Non
linearity.
Slope
negative.
Slope
increases
with YTM.**



CONVEXITY OF PRICE-YIELD CURVE

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+y)^t} \text{ so that}$$

$$\frac{dP_0}{dy} = -\frac{1}{(1+y)} \sum_{t=1}^T \frac{tC_t}{(1+y)^t} < 0;$$

$$\frac{d^2P_0}{dy^2} = +\frac{1}{(1+y)^2} \sum_{t=1}^T \frac{t(t+1)C_t}{(1+y)^t} > 0$$

thereby establishing the convexity of the price - yield curve.

Non linearity.

**First derivative is negative.
Slope is negative, θ is in
second quadrant.**

**Second derivative is positive.
Hence, the curve is convex to
the origin.**

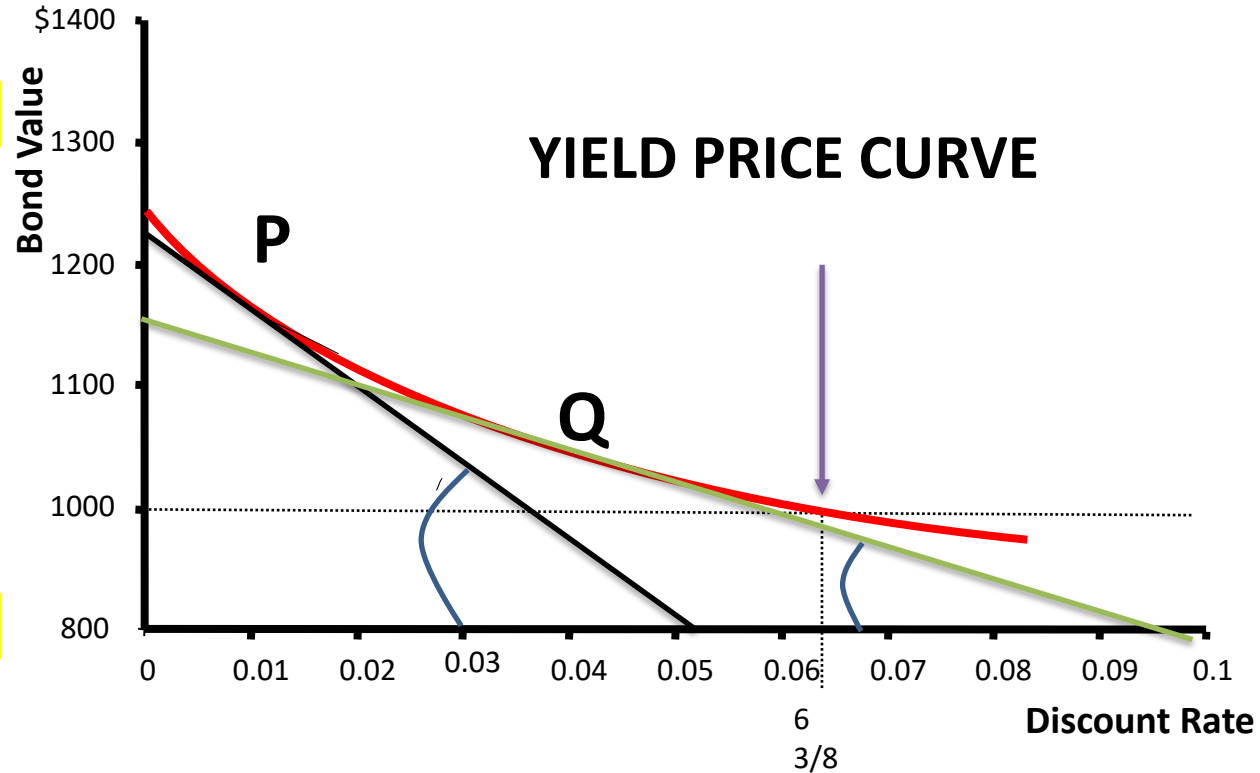


YTM AND BOND PRICE: CONVEXITY

Negative first derivative:
Price and YTM are inversely related.

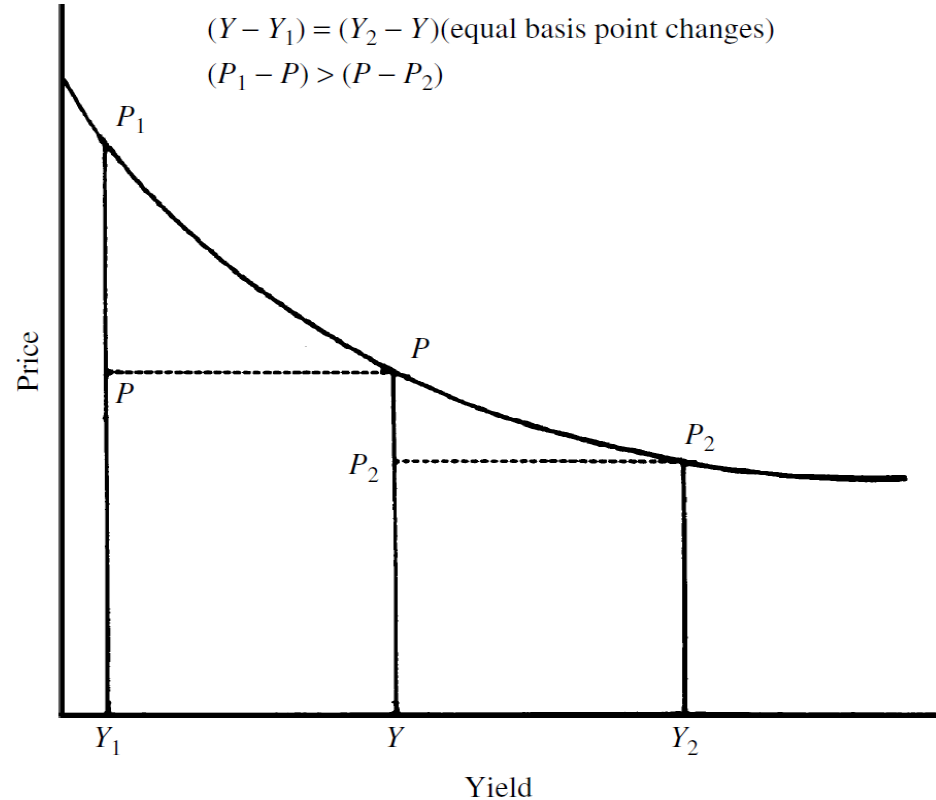
Positive 2nd derivative:
Slope increases with YTM.

These two together imply that the magnitude of the slope is decreasing with YTM thereby establishing convexity.



IMPACT OF CONVEXITY

For a given decrease in yield, the percentage price increase is greater than the percentage price decrease for an equal increase in yield.



MEASURES OF INTEREST RATE RISK

- DOLLAR VALUE PER BASIS POINT (DV01)
- **MACAULAY'S DURATION & CONVEXITY**
- MODIFIED DURATION
- INTEREST RATE ELASTICITY



DOLLAR VALUE PER BP

- **DOLLAR VALUE PER BASIS POINT (DV01) IS THE CHANGE IN BOND PRICE CORRESPONDING TO A CHANGE OF ONE BASIS POINT IN THE YIELD**
- It is given by the negative slope of the price/yield curve:

$$DV01 = -\frac{dP(y)}{dy}$$



MACAULAY'S DURATION

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+y_0)^t}; \quad P_0 = P(y_0); \quad P_0 + dP = P(y_0 + dy)$$

Expanding $P(y_0 + dy)$ as a Taylor series around y_0 , we have

$$P(y_0 + dy) = P(y_0) + P'(y_0)dy + \frac{1}{2}P''(y_0)dy^2 + \dots$$

$$\left. \frac{dP}{P} \right|_{y_0} = \frac{P(y_0 + dy) - P(y_0)}{P(y_0)} = \frac{P'(y_0)}{P(y_0)}dy + \frac{1}{2} \frac{P''(y_0)}{P(y_0)}(dy)^2 + \dots$$

$$\left. \frac{dP}{P} \right|_{y_0} = \frac{P(y_0 + dy) - P(y_0)}{P(y_0)} = \frac{P'(y_0)}{P(y_0)} dy + \frac{1}{2} \frac{P''(y_0)}{P(y_0)} (dy)^2 + \dots$$

$$= -D_{\text{Mac}} \frac{dy}{(1 + y_0)} + C \left(\frac{dy}{1 + y_0} \right)^2$$

$$\text{Duration } D_{\text{Mac}} = - \frac{(1 + y_0) P'(y_0)}{P(y_0)} = \frac{\sum_{t=1}^T \frac{t C_t}{(1 + y_0)^t}}{P(y_0)},$$

$$\text{Convexity } C = \frac{(1 + y_0)^2 P''(y_0)}{2P(y_0)} = \frac{\sum_{t=1}^T \frac{t(t+1) C_t}{(1 + y_0)^t}}{2P(y_0)}$$

DURATION AS DCF WEIGHTED AVERAGE TIME

- A bond's (annual) Macaulay duration is calculated as the **weighted average of:**
- **the number of years until each of the bond's promised cash flows,**
- **where the weights are the present values of each cash flow as a percentage of the bond's full value.**

$$\text{Duration } \mathbf{D}_{\text{Mac}} = \frac{\sum_{t=1}^T \frac{t\mathbf{C}_t}{(1+y_0)^t}}{\mathbf{P}(y_0)}$$

$$= \sum_{t=1}^T t \times \left[\frac{\frac{\mathbf{C}_t}{(1+y_0)^t}}{\mathbf{P}(y_0)} \right] = \sum_{t=1}^T t \times \left[\frac{\frac{\mathbf{C}_t}{(1+y_0)^t}}{\sum_{t=1}^T \frac{\mathbf{C}_t}{(1+y_0)^t}} \right]$$

DURATION OF COUPON BONDS

- **Between coupon dates, the Macaulay duration of a coupon bond decreases with the passage of time and then goes back up significantly at each coupon payment date.**



IF WE IGNORE CONVEXITY, THEN

**DURATION
AS LINEAR
APPROX**

$$\left. \frac{dP}{P} \right|_{y_0} = \frac{P'(y_0)}{P(y_0)} dy = -D_{\text{Mac}} \frac{dy}{(1+y_0)} \quad \text{or} \quad \left. \frac{dP}{dy} \right|_{y_0} = -D_{\text{Mac}} \frac{P(y_0)}{(1+y_0)}$$

$$\text{where } D_{\text{Mac}} = -\frac{(1+y_0)}{P(y_0)} P'(y_0) = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{P(y_0)} \text{ is fixed for given } y_0;$$

$$\text{or } \left. \frac{dP}{dy} \right|_{y_0} = P'(y_0) = \text{constant at given } y_0 \text{ showing that "duration" is a linear}$$

approximation of the yield-price curve around the point of reference.

